

# 2006 年前期 力学 試験問題 解答例.

問 1. (1)  $\dot{\mathbf{e}}_r = (-\sin\theta \hat{x} + \cos\theta \hat{y}) \dot{\theta} = \dot{\theta} \mathbf{e}_\theta$

$\dot{\mathbf{e}}_\theta = -(\cos\theta \hat{x} + \sin\theta \hat{y}) \dot{\theta} = -\dot{\theta} \mathbf{e}_r //$

(2)  $\dot{\mathbf{r}} = \dot{r} \mathbf{e}_r + r \dot{\mathbf{e}}_r = \dot{r} \mathbf{e}_r + r \dot{\theta} \mathbf{e}_\theta$

$\ddot{\mathbf{r}} = \ddot{r} \mathbf{e}_r + \dot{r} \dot{\mathbf{e}}_r + \frac{d}{dt}(r\dot{\theta}) \mathbf{e}_\theta + r\dot{\theta} \dot{\mathbf{e}}_\theta$

$= (\ddot{r} - r\dot{\theta}^2) \mathbf{e}_r + \left\{ \dot{r}\dot{\theta} + \frac{d}{dt}(r\dot{\theta}) \right\} \mathbf{e}_\theta$   
 $= \ddot{r}\dot{\theta} + r\ddot{\theta}$

$\therefore a_r = \ddot{r} - r\dot{\theta}^2, a_\theta = 2\dot{r}\dot{\theta} + r\ddot{\theta} = \frac{1}{r} \frac{d}{dt}(r^2\dot{\theta}) //$

(3) (2) より  $\ddot{r} - r\dot{\theta}^2 = -\frac{GM}{r^2}$

$\frac{1}{r} \frac{d}{dt}(r^2\dot{\theta}) = 0 \rightarrow r^2\dot{\theta} = \text{一定.}$

$\therefore$  411. 面積速度  $(\frac{1}{2}r^2\dot{\theta})$  - 一定 = 47055 - 1 第2法則に等しい.

角運動量は  $L = \mathbf{r} \times \mathbf{p} = m r v_\theta (\mathbf{e}_r \times \mathbf{e}_\theta)$

$= m r^2 \dot{\theta} (\mathbf{e}_r \times \mathbf{e}_\theta)$   
 $v_\theta = r\dot{\theta}$

$\therefore r^2\dot{\theta} = \text{一定}$  は 角運動量保存則に等しい //

(4) 円運動の時  $\dot{r} = \dot{r} = 0$

$\therefore \ddot{r} - r\dot{\theta}^2 = -\frac{GM}{r^2} = -\frac{GM}{r}$  or  $\dot{\theta}^2 = \frac{GM}{r^3} : r = \frac{1}{\dot{\theta}^2}$

$\therefore T^2 = \left(\frac{2\pi}{\dot{\theta}}\right)^2 = (2\pi)^2 \frac{r^3}{GM} \rightarrow 47055 \text{ 第3法則} //$